2.6: Summary

Let's review what we've learned about diffusion and Darcy's law so far.

The Diffusion Equation

Diffusion is how a substance moves from a region of high concentration to a region of low concentration. It can be described using the diffusion equation

\[
\frac{dT}{dt} = \kappa \frac{d^2T}{dz^2}
\]

To solve this PDE so that we can apply it to real problems in geology, we take several steps.

1. We first determine what is the initial condition for the variable we are given?
2. What are the boundary conditions for the variable?
3. What is η in the error function equation?
4. Use the initial conditions and boundary conditions to determine the constants of integration in the general solution

Geological Applications of the Diffusion Equation and Darcy's Law

We applied these PDE solving skills to several problems, the Cooling of a Lithospheric Plate, Fault Scarp Erosion, and Diffusion of a Chemical Species.
We also looked at Darcy's law and how it applies to flow in closed spaces such as pipes and groundwater. Darcy's law can be written as

\[ q = -K \frac{dh}{dx} \quad \text{or} \quad q = -\frac{k}{\mu} \frac{dp}{dx} \]

where \( q \) is Darcy flux, \( k \) is the permeability in \( m^2 \), \( K \) is the hydraulic conductivity, \( \mu \) is the fluid viscosity in Pa·s, and \( \frac{dh}{dx} \) is the hydraulic gradient. It describes how a fluid flows in a porous medium.

On a smaller scale, fluid flow can either be laminar or turbulent. This depends on the fluid's Reynold's number (Re).

\[ Re = \frac{\rho u_o L}{\mu} \quad \text{Reynold's number} \]

where \( u_o \) is flow velocity and \( L \) is fluid depth.

To have Darcy flow, the \( Re < 1-10 \) (laminar) or \( Re > 1-10 \) (non-linear/not-laminar). An \( Re > 1-10 \) may not be laminar flow, but it is not turbulent either. To have turbulent flow, the \( Re > 2000 \).